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Matter effects on Majorana neutrino phases

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ABSTRACT

We consider the effect of ambient matter on the Majorana phase of neutrinos. We find that this can lead to an observable signal if a neutrino oscillation experiment could be performed where the source and the detector are at appropriately different matter densities. We illustrate the situation using a beta beam neutrino source as an example and show that a 5σ signal for the matter modification of the Majorana phase could be possible in a 5-year run.

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I Introduction

CP non-conservation continues to remain an intriguing aspect of particle physics. Since its original observation in the neutral K meson system, further progress has been made more recently through the establishment of direct CP-violation and also its verification in B-meson decays. CP-violation is incorporated in the Standard Model through the Cabibbo, Kobayashi, Maskawa (CKM) [1] mixing among three generations of massive quarks involving one phase angle.

There is no experimental signature of CP-violation in the leptonic sector. The extension of the CKM concept to leptons was precluded by the long-standing belief that neutrinos are massless, permitting mixing to be rotated away. This view has changed in the past decade – results from the atmospheric, solar, reactor, and accelerator neutrino experiments give strong evidence for non-vanishing masses [2]. In this situation, it is but natural to expect CP-violation to show up in leptons as well. This has attained especial importance because the large-scale neutrino experiments being now planned for the future will be geared to permit precision measurements of the neutrino mass and mixing parameters, including CP-violation.

In this work we examine CP phases in the neutrino sector, focusing on two aspects: the additional 'Majorana phases' and the matter induced Mikheev-Smirnov-Wolfenstein (MSW) [3], [4] contribution to these. We show that the matter effects may play an important role in revealing these phases, which are characteristic of Majorana neutrinos.

II Majorana phases

Neutrinos are electrically neutral, so they can be self-conjugate. Such a Majorana neutrino [5] is also absolutely devoid of other charges like lepton number as no quantum number could serve the purpose of differentiating a neutrino from an anti-neutrino. There are several ongoing experiments searching for neutrinoless double beta decay events. Incontrovertible positive evidence here will lend support to the Majorana nature of neutrinos [6].

Majorana neutrinos offer a richer prospect for CP-violation as more phases are permitted for the same number of generations [7], [8], [9]. For example, CP-violation becomes possible even for two lepton generations. These phases have the following origin.

Lepton number is a conserved Noether charge arising from the continuous symmetry:

$$\psi_i \to e^{i\alpha_i}\psi_i \Rightarrow \mathcal{L} \to \mathcal{L}. \tag{1}$$

The index i runs over the generations and ψs are the lepton fields. The transformation in Eq. (1) permits the removal of some phases from the leptonic mixing matrix, as is done for quarks in arriving at the CKM matrix. Majorana nature of the neutrinos would imply the absence of this symmetry, thereby introducing additional observable phases,

apart from the usual neutrino masses and mixing angles. For n generations of neutrinos, there will be $n-1$ such additional observables, since only phase differences have physical significance [10]. In principle, these can be measured in neutrino experiments. In this paper we consider two generations, though the basic argument can be readily extended. The additional observable phases, often termed Majorana phases, are CP odd, that is, they change sign under CP transformation. The phase differences show up in CP asymmetries of processes which, in the simplest cases, involve at least two different amplitudes. These amplitudes each carry a CP even and a CP odd phase and the difference between the two CP even (CP odd) phases must be non-zero, e.g.,

$$A = A_1 e^{i(\gamma_1+\beta_1)} + A_2 e^{i(\gamma_2+\beta_2)}, \tag{2}$$

where γ_i are CP even and β_i are CP odd (i=1,2) and further $\gamma_1 \neq \gamma_2$, $\beta_1 \neq \beta_2$.
Neutrinoless double beta decay and neutrino-anti-neutrino oscillations are examples of processes where these Majorana phases appear [10], [11]. Of these, the former is being looked for in several experiments [6]. However, it has been noted [12], [13], [14], [15] that the present constraints on neutrino masses and mixings make it unlikely for the Majorana phases to be determined through this route. A process where the Majorana phases can be measured, possibly only in principle, is the lepton number violating phenomenon loosely termed as neutrino anti-neutrino oscillation [10]. Here, a massive left-handed neutrino, emitted in a beta decay along with a charged anti-lepton, undergoes a chirality flip and is detected in an inverse beta decay. In this process a charged anti-lepton is emitted during detection, as against a charged lepton in neutrino oscillations. The probability of such a process, symbolically denoted by $P(\nu_e \rightarrow \bar{\nu}_\mu)$, is [16]

$$P(\nu_e \rightarrow \bar{\nu}_\mu) \;\;=\;\; \left(\frac{\sin 2\theta}{2E}\right)^2 \left[m_1^2 + m_2^2 - 2m_1 m_2 \cos\left(\frac{\Delta m_{12}^2}{2E}t - \alpha\right)\right], \tag{3}$$

where, the neutrino flavour eigenstates are indicated by ν_e and ν_μ for simplicity of notation, though the equation is valid for any two generations. The energy of the neutrinos is E, the mass squared difference $\Delta m_{12}^2 = m_2^2 - m_1^2$, and θ is the mixing angle. α ($\equiv (\beta_1 - \beta_2)$ of Eq. (2)) is the CP-violating observable Majorana phase difference[1]. Eq. (3) gives the probability of a chirality flip together with flavour oscillation, given the mass eigenvalues m_i and the mixing angle θ. A similar equation can be written for the probability of chirality flip alone. Neither of these phenomena is truly viable from the point of view of present-day experiments due to the helicity suppression factors, $\frac{m_i}{E}$, in Eq. (3).
It should be noted that in the two generation case, if either one of the mass eigenvalues is vanishing then the Majorana phase can be removed. Unless stated otherwise, the mass eigenvalues are assumed to be non-zero.

[1] $(\frac{\Delta m_{12}^2}{2E}t) \equiv (\gamma_1 - \gamma_2)$ of Eq. (2).

III The MSW mechanism and Majorana Phases

The Majorana phase cannot induce any CP asymmetry in vacuum neutrino oscillations. To explore the situation in the presence of ambient matter, we first summarize the MSW mechanism in some detail, keeping in mind the Majorana property of the neutrinos.

In terms of a conventional neutrino state ν and its charge conjugate ν^c, a Majorana neutrino is[2] $N \equiv (\nu \pm \nu^c)/\sqrt{2}$. One of these combinations is assumed to be light and is the focus of our attention. In see-saw models of neutrino mass, the other is heavy and we do not consider it any further. In discussing the matter interactions of such a Majorana neutrino, N, one has to bear in mind that its left chiral projection is a superposition of a member of an $SU(2)_L$ doublet (ν_L) and a sterile state (ν_L^c). Below we refer to N_L as the *neutrino* (ν) to indicate the active component. By the same token, in N_R only the ν_R^c participates in weak interactions. We refer to it as the *anti-neutrino* ($\bar{\nu}$) in the following.

Neutral current interactions are the same for all active neutrinos and do not affect neutrino oscillations. The effective propagator of electron neutrinos is modified due to charge current interactions with the ambient matter. Ignoring mixing for the moment, the matter-modified energy for spin $\frac{1}{2}$ electron neutrinos can be written as[3],

$$E = \alpha.\mathbf{p} + \beta m \rightarrow E = \alpha.\mathbf{p} + \beta m + \sqrt{2} G_F n_e, \tag{4}$$

which follows from [17], [18], [19],

$$G(p) = \frac{i}{\not{p} - m} \rightarrow \frac{i}{\not{p} - m - \gamma^0 \sqrt{2} G_F n_e} \quad . \tag{5}$$

The above result, obtained using finite temperature field theory methods, is valid in the rest frame of the ambient matter. For a Majorana neutrino, the propagator contains additional terms involving chirality flip. However, these terms will not be important for the subsequent discussion.

For Majorana neutrinos, the particle cannot be distinguished from the antiparticle. CP-violation due to the Majorana phase α is a characteristic of the spin-half nature of the neutrinos. This is evident from the vital role played by chirality flip in the CP-violating neutrino-anti-neutrino oscillation phenomenon. This is unlike CP-violation due to a Dirac phase, which, in principle, could also be formulated for spin zero particles.

The effect of mixing can be incorporated by promoting m to a (2×2) matrix M_ν. We choose to work in a basis in which the charged lepton mass matrix is diagonal and where the Majorana phase is included as a multiplicative factor associated with the neutrino fields. Due to the Majorana property, these phases cannot be absorbed into

[2]More generally, $N \equiv (e^{i\phi}\nu \pm e^{-i\phi}\nu^c)/\sqrt{2}$.

[3]On the RHS of the arrow are the matter-modified quantities hereafter.

the neutrino fields and will have observable consequences. In this basis, the neutrino mass matrix, M_ν, and the mixing matrix, U, are real.
In the highly relativistic limit, the effect of ambient matter on the neutrino mass matrix can be written as [3], [4],

$$M_\nu M_\nu^\dagger \to M_\nu M_\nu^\dagger + \begin{pmatrix} 2\sqrt{2}G_F n_e E & 0 \\ 0 & 0 \end{pmatrix}, \tag{6}$$

in the flavour basis. The second term on the RHS of the arrow – the matter modification – takes a negative sign for anti-neutrinos. The eigenvalues of the RHS matrix are the squared mass eigenvalues in the medium, whereas the two eigenvectors determine the diagonalizing matrix and hence, the matter-modified mixing angle θ_m. The diagonalizing matrix is the same for Dirac as well as Majorana neutrinos [20]. It should be noted that this matrix is not unique. There is freedom of adding a phase common to a row or a column or both. The most general matter-modified mixing matrix can be parametrized as,

$$U_m = \begin{pmatrix} \cos\theta_m e^{i\eta_1} & \sin\theta_m e^{i\eta_1+i\eta_2} \\ -\sin\theta_m & \cos\theta_m e^{i\eta_2} \end{pmatrix}. \tag{7}$$

The phases η_1 and η_2 are undetermined from the mass matrix and cannot be fixed from the physics under consideration. To simplify matters, it would not be unreasonable to choose them to be independent of the mixing angles θ_m. It then follows that both η_i have to be zero in order to ensure that in the limit of ambient matter density going to zero, the vacuum mixing matrix, U, is reproduced, where

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \tag{8}$$

The matter-modified eigenstates of the neutrinos can be expressed in terms of the vacuum mass eigenstates as

$$\begin{pmatrix} e^{i\alpha_1}\nu_{1m} \\ e^{i\alpha_2}\nu_{2m} \end{pmatrix} = U_m^\dagger \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = U_m^\dagger U \begin{pmatrix} \nu_1 \\ e^{i\alpha}\nu_2 \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \nu_1 \\ e^{i\alpha}\nu_2 \end{pmatrix}. \tag{9}$$

where $\phi = \theta_m - \theta$.
The phases can be readily found to be

$$\tan\alpha_1 = \frac{\sin\phi\sin\alpha}{\cos\phi + \sin\phi\cos\alpha}, \tag{10}$$

$$\tan\alpha_2 = \frac{\cos\phi\sin\alpha}{\cos\phi\cos\alpha - \sin\phi}. \tag{11}$$

It should be noted that in matter the relative Majorana phase, responsible for CP-violation, is:

$$\begin{aligned} \alpha' &= \alpha_2 - \alpha_1 \\ &= \tan^{-1}\left(\frac{\cos\phi\sin\alpha}{\cos\phi\cos\alpha - \sin\phi}\right) - \tan^{-1}\left(\frac{\sin\phi\sin\alpha}{\cos\phi + \sin\phi\cos\alpha}\right). \end{aligned} \tag{12}$$

It vanishes when the vacuum Majorana phase $\alpha = 0$. Recall that $\phi = \theta_m - \theta$ and therefore α' is significantly different from α near a resonance.

IV CP asymmetries in neutrino oscillations

A consequence of the propagation eigenstates not being the same as flavour eigenstates is the phenomenon of neutrino oscillations. Oscillation of neutrino flavours requires a nontrivial mixing angle θ and non-degenerate mass eigenvalues of the neutrino mass matrix. The probability amplitude of neutrino oscillations,

$$A(\nu_e \to \nu_\mu) = \sum_i \langle \mu^- | \gamma^\mu (1-\gamma_5) U_{\mu i} | \nu_i(t) \rangle \langle \nu_i(0) | U_{ei}^* \gamma_\mu (1-\gamma_5) | e^- \rangle \tag{13}$$

is modified due to the presence of ambient matter. Matter effects entail the following changes for Majorana neutrinos:

$$\begin{aligned} \Delta m_{12}^2 &\to \sqrt{(\Delta m_{12}^2 \cos\theta - 2\sqrt{2} G_F n_e E)^2 + (\Delta m_{12}^2 \sin\theta)^2} \;, && (14) \\ \tan 2\theta_m &\to \frac{\sin 2\theta}{\cos 2\theta - 2\sqrt{2} G_F n_e E} \;, && (15) \\ \alpha &\to \alpha'. && (16) \end{aligned}$$

Apart from the usual modifications made for Dirac neutrinos, there is a further replacement given by Eq. (16), where α' is given in Eq. (12).
In the adiabatic limit, the probability of neutrino oscillations in matter for the two-flavour case is given by

$$P(\nu_e, 0 \to \nu_\mu, t) = \left| U_{e1}^*(0) U_{\mu 1}(t) e^{i\int E_1(t) dt - i\alpha_1} + U_{e2}^*(0) U_{\mu 2}(t) e^{i\int E_2(t) dt + i\alpha - i\alpha_2} \right|^2 . \tag{17}$$

The dependence of the effective energy and the mixing angle on time is a result of the varying matter density. It should be noted that the Majorana phase, like the mixing matrix, is not a kinematical quantity. It plays a role only at emission and detection and hence, unlike the effective energy, is not integrated over time.
Assuming for simplicity that the point of emission is in vacuum, the oscillation probability is

$$\begin{aligned} P &= \cos^2\theta \sin^2\theta_m + \sin^2\theta \cos^2\theta_m \\ &- 2 \frac{\sin 2\theta}{2} \frac{\sin 2\theta_m}{2} \cos\left(\int_0^t \frac{m_{12m}^2}{2E} dt + (\alpha - \alpha') \right). \end{aligned} \tag{18}$$

It is seen that the dependence on $(\alpha - \alpha')$ vanishes if the point of detection is also in vacuum. The same result holds even in the case of identical matter densities at

the point of emission and detection, including the special case of uniform matter density. Different densities at the source and detection points is therefore a must for the Majorana phase to be effective in this process.
Using the CP odd nature of α, we arrive at an expression for the CP asymmetry:

$$\Delta P = |P - \overline{P}| = \sin 2\theta \sin 2\theta_m \sin\left(\int_0^t \frac{m_{12m}^2}{2E} dt\right) \sin(\alpha - \alpha'). \tag{19}$$

Here $\overline{P}$ is the oscillation probability of anti-neutrinos.

V Numerical estimates

Phenomenologically, it is the CP asymmetry of a process that can give evidence of CP-violation in a system. However, in the case of neutrinos propagating through matter, the asymmetry,

$$\Delta P = P(\nu_e, 0; \nu_\mu, L) - P(\bar{\nu}_e, 0; \bar{\nu}_\mu, L), \tag{20}$$

gets contribution from CP-violation as well as the overwhelming dominance of ambient matter over antimatter. This implies that the CP asymmetry given by Eq. (19) will need a modification to accommodate the fact that the value of θ_m will be different for neutrinos and anti-neutrinos. Hence, the study of CP-violation through the CP asymmetry would require a disentangling of the contribution due to matter effects and is not the most favourable way of analytically probing the physics we are after.
Therefore, we turn to the following alternative. As noted earlier, $\alpha' - \alpha$ is appreciable only at certain energies – near a resonance – and that too for either neutrinos or anti-neutrinos depending on whether the mass-squared splitting is positive or negative. Given a density profile, for appropriate energies the oscillation probability will then deviate from the one estimated without taking the Majorana phases into account (i.e., α = 0). To experimentally look for this phase it will be useful to have a beam of neutrinos of well-defined energy spectrum. As an illustration, we consider neutrinos from a beta beam source[4] [21], [22]. We consider the ν_e ($\bar{\nu}_e$) survival probability and examine its sensitivity to matter effects through α. The expectation for the number of events for a 5-year run has been illustrated in Fig. 1 for a beta beam source at CERN with a 440kT Water Cerenkov detector[5] at Gran Sasso. In case of Water Cerenkov detectors (e.g., Super-Kamiokande) energy resolution better than 50 MeV can be achieved, which is assumed in our analysis. Here $\gamma = 580$ ($\gamma = 350$) has been chosen and 5×10^{11} (10^{13}) decays per second have been assumed for the neutrino (anti-neutrino) beam.

[4] A beta beam is a collimated neutrino source produced through the decay of accelerated beta-active nuclei.

[5] 100% efficiency has been taken.

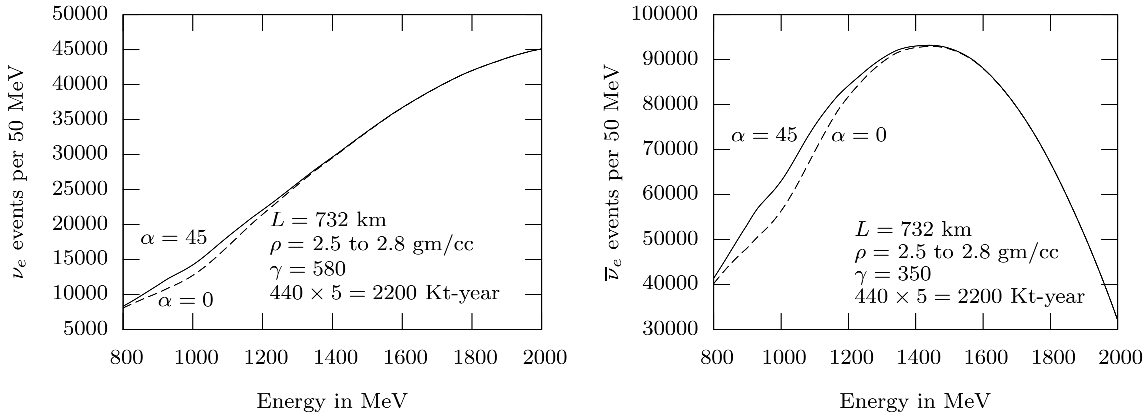

Figure 1: ν_e and $\bar{\nu}_e$ event rates for Majorana phase $\alpha = 45^\circ$ (solid) and $\alpha = 0^\circ$ (dashed) for a 5-year beta beam run. A linear increase of the matter density from 2.5 gm/cc to 2.8 gm/cc over 732 km is assumed. $\Delta m^2 = +2 \times 10^{-3}$ eV^2 (left panel), $\Delta m^2 = -2 \times 10^{-3}$ eV^2 (right panel) and $\theta = 5^\circ$ are used.

The matter density is taken to vary linearly[6] from 2.5 gm/cc to 2.8 gm/cc. We use the mass splittings favoured by the solar and atmospheric neutrino experiments and find that a matter induced enhancement of α is possible only for the ν_e ($\bar{\nu}_e$) to ν_τ ($\bar{\nu}_\tau$) transition for normal (inverted) hierarchy. Under these circumstances, it is justified to use the two-flavour oscillation formula, as $P(\nu_e \to \nu_\mu) \ll P(\nu_e \to \nu_\tau)$. The oscillation probability, needless to say, depends on the value of θ – which we have chosen to be 5°; well within the experimental bound for θ_{13} – and may vanish for a particular combination of θ and L. The modification in event rate is prominent at energies which are not enough to produce tau leptons. However, the effect of the Majorana phase should be detectable at more than 5σ level (for $\alpha = 45^\circ$) in a disappearance experiment of electron neutrinos once the other mixing angles and mass-splittings are measured to sufficient precision. Note that the deviation in flux due to α is appreciable only if the matter densities at the point of emission and detection are close to the resonance value. For a given vacuum mixing angle, to observe this effect at higher energies the matter densities would have to be lower and vice versa. In the example displayed in Fig. 1, there is no impact of the Majorana phase at high energies. In fact, such an energy dependence can be considered as an evidence for the Majorana property of the neutrinos, though the absence of such an effect would not demonstrate the reverse.

[6]The densities chosen for this example are close to the matter density of the earth but the linear variation does not reflect the true profile.

VI Conclusions

The possibility of observing the Majorana phase in neutrino oscillations through a matter-induced effect may be difficult due to the practical limitation of the availability of a suitable density profile. Nonetheless, it may not be out of reach of proposed precision experiments. Oscillation of neutrinos in the presence of ambient matter may thus play an important role in revealing the Majorana nature of neutrinos.

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